Synchronization modes of the mechanical response in mouse heart

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***Short Abstract* — The heart rhythm as a system of weakly coupled oscillators was studied recently, but little attention has been paid to the synchronization modes of the electrical stimulation-contractile response coupling in isolated mouse heart. This study aimed to describe with a minimalistic mathematical model the synchronization modes on the electrical stimulation-contractile response coupling in the whole heart. We propose a minimal linear coupled oscillator model to study the synchronization modes, which is validated with experimental results. The local stability is studied. We predict through in-silico experiments the presence of several synchronization modes, and these could be associated with arrhythmias.**

***Keywords* — Linear model, synchronization modes, systems biology, stability and complex dynamics, modeling and identification of nonlinear systems.**

I. Introduction

THE cardiovascular system has been studied with a substantial body of mechanistic mathematical models [1–5].

Christie et al., recently modeled the cardiovascular system as a series of weakly coupled oscillators. The interactions between these oscillators generate a chaotic blood pressure waveform signal. A minimal linear model to identify different dynamical scenarios as a function of the parameters values is presented [6]. However, although the interaction between heart rate, sympathetic nervous, parasympathetic nervous, and respiratory systems were analyzed, little attention has been paid to the synchronization modes in electrical stimulation-contractile response coupling.

In this paper, we propose a minimal linear mathematical model of the electrical stimulation-contractile response coupling to address the following question: whether the synchronization modes can be obtained with a model of weakly coupled oscillators. Using stability analysis, we found a stable equilibrium point to the proposed linear model. Then, we performed simulations of the coupled model with a forced function that represents electrical stimulus.

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II. Results

The model shown in (1) where k_C is the coupling parameter, ω_E is the sinoatrial node frequency, ω_M is the final contraction frequency, $f(t)$ is the external electrical stimulation function which was chosen as a square signal with 10% of the duty cycle.

$$\begin{aligned} \dot{Z}_1 &= Z_2, \\ \dot{Z}_2 &= f(t) - (\omega_E{}^2 + k_c)\, Z_1 + k_c\, Z_3, \\ \dot{Z}_3 &= Z_4, \\ \dot{Z}_4 &= k_c\, Z_1 - (\omega_M{}^2 + k_c)\, Z_3, \end{aligned} \tag{1}$$

We present five cases that describe five synchronization modes for different parameter settings:

Parameter	Value				
Mode	**1:1**	**2:1**	**1:2**	**3:2**	**2:3**
$\omega_E\ [rad/s]$	37.7				
$\omega_M\ [rad/s]$	37.7				
$k_C\ [Hz^2]$	900	260	3157	1280	3680
$Z_{1\ o}\ [A.U.]$	0.1				
$Z_{2\ o}\ [A.U.]$	0.5	1.2	0.4	3.5	3.5
f(t) amplitude $[A.U.]$	16000	7020	3000	4500	390
f(t) freq $[Hz]$	9	14	7	15	10

III. Conclusion

The simplicity of the model precludes it from capturing the molecular events at the intracellular level. We have illustrated how changes in the coupling parameter control the synchronization modes. The model reproduces 1:1, 1:2 and 2:1 synchronization modes, experimentally obtained. The main prediction of the model is that 2:3 and 3:2 synchronization modes can be obtained changing the coupling parameter.

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