Problems of Type Theory and Regularization for Morphodynamics

Eric Mjolsness[1]

***Short Abstract* — Biological modeling can be aided by computational tools, at the cost of formalization. For example, biological modeling languages can be built up in terms of process models each of which has a defined time-evolution operator. Extending this approach to geometrical objects such as tissues, membranes, or cytoskeleton brings up technical problems in regularization and also in type theory.**

***Keywords* — Morphodynamics, development, biomechanics, regularization, type theory, moldeling languages.**

I. Introduction

PROCESS modeling languages, applicable to richly complex biological systems, can be defined in terms of time-evolution operators for their separate processes [1]. For processes acting simultaneously in continuous time, the operators simply add or integrate up. The collections of objects acted on must therefore be indexed by a measure space. But what if the objects are themselves continuous, such as surface representations of membranes or the dynamic tissue morphologies of biological development? Then a functional integral [2] must be regularized. What if such functions of space and time are further acted on by a selective environment during evolution? Could an entire functional from spacetime behavior to reproductive success be modeled as a dynamical object? Such a conceptual approach might require a new level of regularization.

Type theory was introduced into the foundations of mathematics by Russell to resolve paradoxes of unrestricted set theory, and has developed into a theoretical framework for programming languages. Type theory allows one to construct new types of mathematical objects (such as object tuples or disjoint unions) out of old ones, using type constructors such as Cartesian products (tuples), disjoint sums (unions), and higher-order functions. Thus, a set of allowed functions from type 1 to type 2 might comprise type 3. For example continuous objects like surfaces or s deformations of surfaces consist of continuous functions between (at least) topological spaces. However, repeated type construction by functions has the potential to increase the cardinality of the resulting types, and/or to destroy whatever property of measurability is needed for operator integration. Again some kind regularization is required.

Acknowledgements: Funded by NIGMS grant R01 GM086883.
[1]Department of Computer Science, University of California, Irvine. emj@uci.edu.

II. Approaches

We discuss two approaches to the problems higher-order functional integrals for morphodynamic modeling.

A potentially very general approach is top-down: Seek a category (literally) of mathematical objects that is closed under products and higher-order functions (called Cartesian Closed Categories or CCCs), and yet amenable to some use of measure theory. A current trend in programming language theory seeks to exploit topological CCCs such as the category of compactly generated topological spaces (too general for our applied mathematical purposes) among others. Success in this approach would imply a solution to the open problem of finding a CCC whose objects all permit the formation of measure spaces; probably some restriction of this goal is needed for the top-down approach to succeed.

An alternative, bottom-up approach is to (a) define regularization of necessary functional integrals and (b) to rule out higher order functions and integration over them. The minimally necessary functional integrals over operators for (a) are those that would support the definition of classical PDEs such as diffusion, active transport, and elasticity theory. I discuss the use of Sobolev regularizers for this purpose, and relate them to finite element numerical methods. It is also necessary to state the set of function domain geometries required for biological applications; I argue that the whole inclusion hierarchy $R^{d\leq 3} \hookrightarrow \text{dmanifolds} \hookrightarrow \text{cell complexes} \hookrightarrow \text{stratified spaces}$ is necessary conceptually, though adaptively-finite piecewise polynomial approximations to each geometry are all that we need to compute. To formalize condition (b), one can formulate special-purpose type theories that allow only some function-type productions, cutting them off at some low level in the hierarchy of function types.

III. Conclusion

Disparate theoretical frameworks from logic (type theory) and field theory (regularization and functional integrals) may be required to formalize morphodynamic modeling languages for physical biology. Top-down and bottom-up approaches through the labyrinth are sketched.

References

[1] Mjolsness E. and Yosiphon G. (2006). "Stochastic Process Semantics for Dynamical Grammars", Annals of Mathematics and Artificial Intelligence, 47(3-4) August 2006.

[2] Mjolsness E. (2010). "Towards Measurable Types for Dynamical Process Modeling Languages". Electronic Notes in Theoretical Computer Science, vol. 265, pp. 123-144, 6 Sept. 2010, Elsevier.