Predictive information and population dynamics in a changing world

Quentin Feltgen[1] and Ilya Nemenman[2]

Short Abstract — **Substantial theoretical population dynamics work has focused on determining which strategy an individual should follow to maximize its growth rate in a fluctuating environment. Here we investigate dynamics of a population that gradually "learns" and adapts its response strategy to an observed statistics of a changing environment. We observe that this leads to sublinear corrections to the population growth rate, which are nearly universal in structure and are equal to the predictive information in the environment time series, which in its turn is a characterization of time series complexity.**

Keywords — **learning, adaptation, predictive information, population dynamics**

I. Introduction

Living organisms experience an ever-changing world, and their behavioral strategies must be adjusted constantly to track environmental changes and optimize their fitness, measured by population growth rate. One type of environments and response strategies that have been understood particularly well is the switch problem, where environment switches randomly among discrete states (e.g., presence or absence of antibiotics), and organisms respond by epigenetically selecting one of available phenotypes (e.g., persistence or growth), which each have different growth rates in either environment [1]. It is understood that stochastic phenotype switching realizes a bet-hedging strategy, and the population growth can be maximized in certain cases without costly sensory mechanisms by matching phenotypic switching rates to those of the environment [2-4]. Crucially, the optimal long-term population growth rate in these problems is closely related to the amount of information an individual has about the environment [2-4]. Unfortunately, the optimal growth rate and the strategy realizing it depend strongly on the assumptions made about the statistics of the environment and the repertoire of possible strategies, which has hindered experimental validation of the theories.

II. Methodology

Here we notice that statistics of the environment may change with time and are generally unknown to an individual (e.g., frequency of antibiotics may depend on slowly changing factors). Therefore, one can ask: how quickly will a population grow, on average, while adapting its response strategies to a change in statistics of the environment? The growth rate is limited by the ability to learn the statistics from observations of the world. Thus we are interested in how quickly the population will approach its optimal, terminal growth rate.

We consider two scenarios. First, an individual is capable of adapting its behavioral strategy based on memorized experiences. The population then consists of homogeneous individuals, each becoming more fit with every generation. Second, the population may be heterogeneous, and then its growth rate changes by natural selection amplifying frequencies of more fit individuals. In both cases, as time goes by, either individuals or the population as a whole "learn" the statistics of the world more accurately, and the response strategy approaches the optimal one. Thus the problem reduces to estimating the amount of information that the individuals gain about the environment from observations, and we calculate the growth rate by techniques of [5].

III. Results and Conclusion

In all cases, we find that the asymptotic behavior of the population size as a function of time, $N(T)$, averaged over all possible histories of environments and responses, satisfies: $\ln N(T) = \Lambda_0 T - \frac{K}{2}\ln T + O(T^0)$. Here Λ_0 is the optimal growth rate, which is specific to the problem, and K depends on (finite) dimensionalities of the spaces of possible environmental models and response strategies. In the aforementioned phenotypic switch problem, where the response strategy is described by one parameter (probability of switching into one of the two phenotypes), K=1. Unlike the leading term, the logarithmic correction is sublinear in time, so that the population eventually reaches its optimal growth rate. The sublinear term is exactly the predictive information [5] associated with learning a time series parameterized by K parameters. This tight relation between population dynamics, learning theory, and information theory is noteworthy since the sublinear term is largely universal and independent of specific details of the adaptive mechanism, or of the specific distribution of the environmental states. Thus it may be easier to observe experimentally.

Acknowledgements: This research was partially supported by the James S McDonnell Foundation grant No.220020321.

[1]ICFP, Département de Physique, Ecole Normale Supérieure, Paris, France; and Department of Physics, Emory University, Atlanta, GA, USA. E-mail: feltgen@clipper.ens.fr.

[2]Departments of Physics and Biology, Emory University, Atlanta GA, USA. E-mail: ilya.nemenman@emory.edu.

References

[1] Balaban N et al. (2004). *Science* **305**, 1622.
[2] Bergstrom C and Lachmann M (2005) *IEEE Inf Thy Workshop,* 50.
[3] Kussel E and Leibler S (2006). *Science* **309**, 2075.
[4] Rivoire O and Leibler S (2011). *J Stat Phys* **142**, 1124.
[5] Bialek W et al. (2001). *Neural Comput* **13**, 2409.