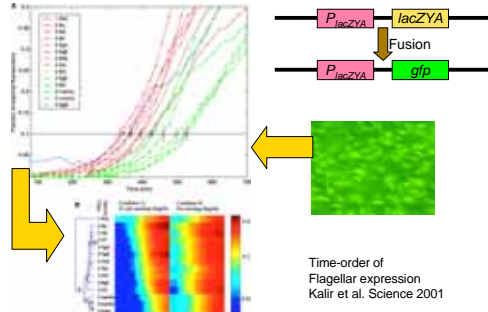
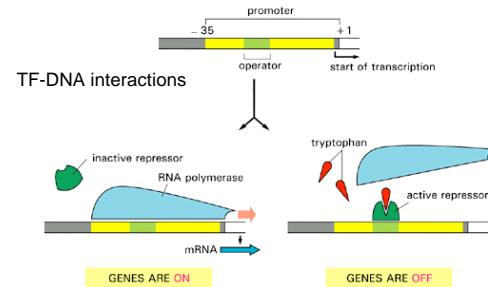


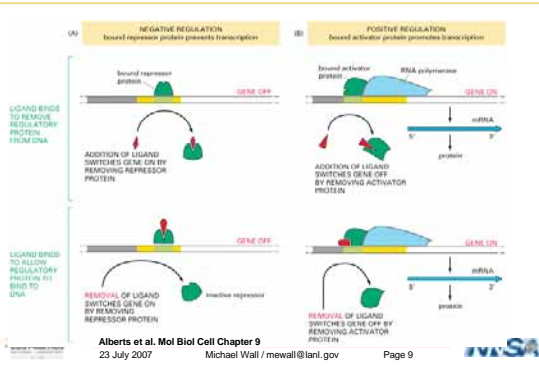
Reporter Assays



Mechanisms of Gene Regulation

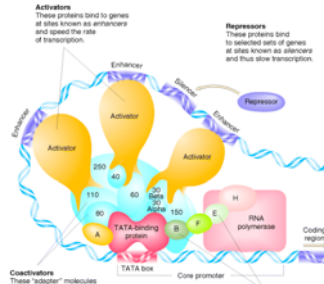


Mode of Control



Mechanisms of Gene Regulation Eukaryotes

- Transcription factors bind to upstream regulatory regions of DNA and communicate the "regulatory logic" to the RNA polymerase
- Chromosome reorganization
- DNA confined to nuclear envelope



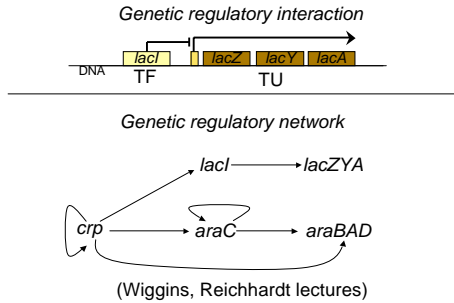
Gene Regulation Themes in q-bio Summer School Session

- Protein levels are regulated by control of transcription
 - Determine mechanisms of transcription and transcriptional control
 - Mike Wall poster
 - Determine mechanisms of translation and translational control
 - Kevin Sanbonmatsu
- Genetic regulatory circuits adjust gene expression in response to specific environmental signals
 - Identify genetic regulatory circuits
 - Chris Wiggins
- The overall genetic regulatory response depends on the detailed structure of a gene circuit
 - Study structure-function relations
 - Cynthia Olson Reichhardt, Mike Wall
- Gene-circuit performance may depend on gene-circuit design
 - Perform mathematically controlled comparisons
 - Mike Wall
- Does evolution select for designs that optimize performance?
 - Elucidate design principles
 - Mike Wall

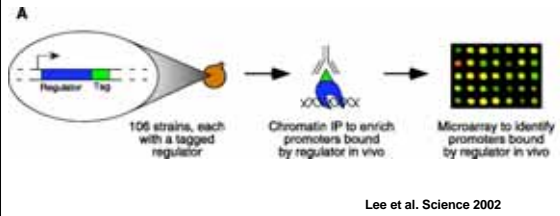
Contents

- Gene regulation
 - Genetic regulatory networks
 - Genetic regulatory circuits
 - Patterns
 - Design principles
 - Mathematical models
 - Controlled mathematical comparisons
 - Structure-function relations

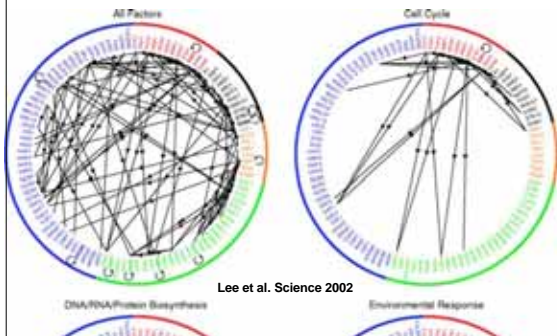
Genetic Regulatory Networks



Genetic Regulatory Interactions Genome-Wide Location Analysis

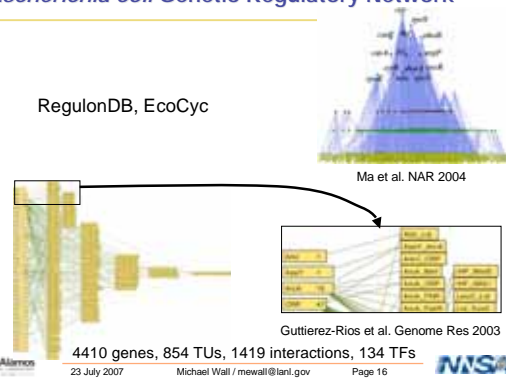


Yeast Genetic Regulatory Network



Escherichia coli Genetic Regulatory Network

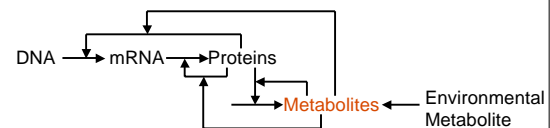
RegulonDB, EcoCyc



Contents

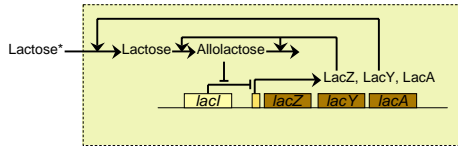
- Gene regulation
- Genetic regulatory networks
- Genetic regulatory circuits
- Patterns
- Design principles
- Mathematical models
- Controlled mathematical comparisons
- Structure-function relations

Signal-Dependent Gene Regulation Genetic Regulatory Circuits



Understanding the functions of genetic regulatory networks will require knowledge of signal interactions

Example: *lac* in *E. coli*

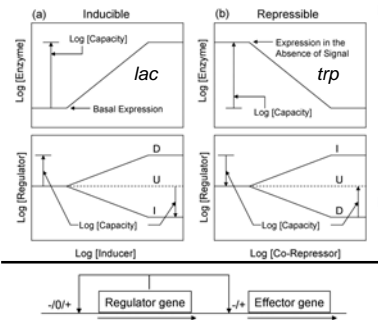


- Inducible catabolic
- Environmental signal is lactose
- *lac* signal molecule is allolactose
- Gratuitous inducers
 - IPTG
 - TMG

Elementary Gene Circuits

- Elementary gene circuits involve just one transcription factor (TF), e.g., LacI
- The TF may regulate its own expression
- TF activity is modulated by a signal
- In bacteria, many circuits fall into two classes
 - Inducible catabolic
 - Produce catabolic enzymes when a key substrate is abundant
 - *lac*
 - Repressible biosynthetic
 - Produce anabolic enzymes when an end product is scarce
 - *trp*

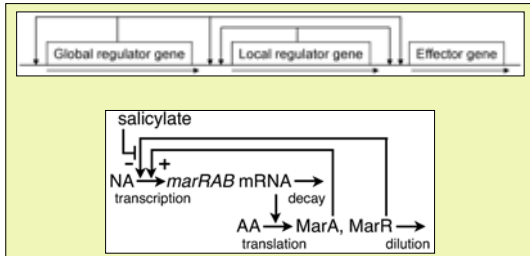
Classes of Elementary Gene Circuits



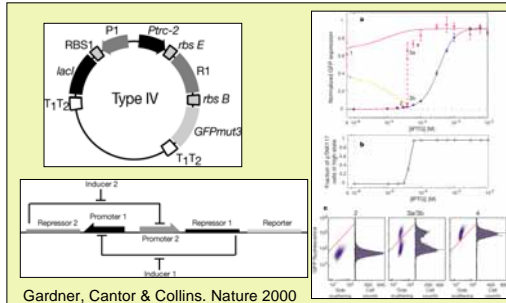
- Function
 - Inducible vs. repressible
 - Logarithmic Gain
 - Expression Capacity
 - Coupling Type (DUI)
- Design
 - Activator vs. Repressor Control
 - Signal-TF Interaction

Binary Gene Circuits

- Two TFs
- Not to be confused with boolean circuit

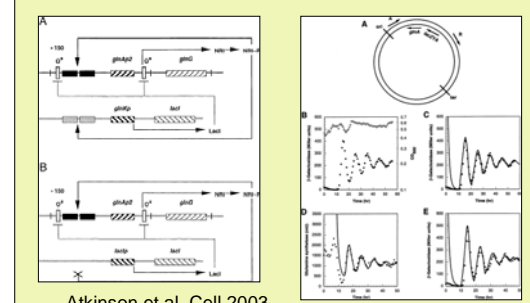


Synthetic Gene Circuits Toggle Switch



Gardner, Cantor & Collins. Nature 2000

Synthetic Gene Circuits Oscillator/Toggle Switch



Atkinson et al. Cell 2003

Engineered Gene Circuits Practical Applications

- **Bioremediation**
 - Active biological containment (ABC) circuit (Ramos 2001)
 - Pm::asd / (Pm::lacI, xyIS, lac::gef) system in a Dasd strain of *P. putida*
- **Biosensor technology**
 - Toxin-induced fluorescent circuit (Bechor 2002)
 - fabA::lux fusion plasmid in *E. coli*
- **Gene therapy**
 - Cancer-specific viral circuit (Ramachandra 2001)
 - Engineered virus that is repressed at normal p53 levels
- **Metabolic engineering**
 - Environment-sensitive metabolic circuit (Farmer & Liao 2000)
 - ACP-induced lycopene production in *E. coli* using a modified Ntr regulon

Studies of Gene-Circuit Design

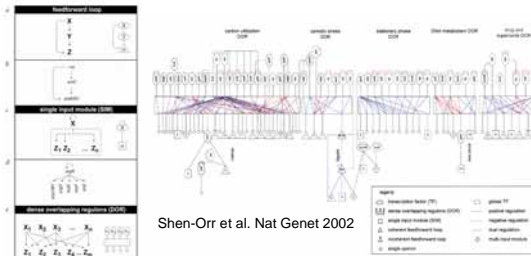


- Understand mechanisms
 - Develop models to study structure-function relations
- Understand the relation between design and performance
 - Perform mathematically controlled comparisons
- Demonstrate understanding through practical applications
 - "Tinker" with Natural and synthetic systems
- Discover and explain patterns in Natural designs
 - Characterize, document, and analyze features of Natural systems

Contents

- Gene regulation
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- Genetic regulatory circuits
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- Controlled mathematical comparisons
- Structure-function relations

Patterns in Gene Regulation Network Motifs in *E. coli*



Shen-Orr et al. Nat Genet 2002

Perhaps these network motifs have distinct information processing roles...but where is the input signal?

Examples of *E. coli* FFLs

Project

(Mangan & Alon, 2003)

Results of counting FFL types:

types: [1,2,3,4]

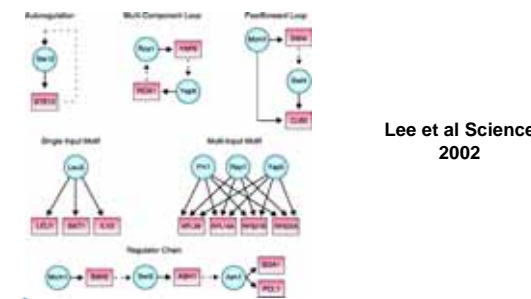
Coherent 28 2 4 1

Incoherent 5 0 1 1

<http://www.weizmann.ac.il/mcb/UriAlon/>

Also see
<http://ecotfs.lanl.gov/FFLs.html>

Patterns in Gene Regulation Network Motifs in Yeast



Lee et al Science 2002

Patterns in Gene Regulation Mode of Control

- Inducible systems
 - Repressor control
 - When substrate is seldom abundant
 - Activator control
 - When substrate is often abundant
- Repressible systems
 - Repressor control
 - When end product is often abundant
 - Activator control
 - When end product is seldom abundant

Savageau PNAS 1977

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Patterns in Gene Regulation Mode of Control, cont'd

	(-) at Regulator TU			(+) at Regulator TU			TF does not control Regulator expression
	I	U	D	I	U	D	
Inducible (+)	4 ^b	3 ^c	4 ^d	0	0	5 ^e	4 ^f
Inducible (-)	0	0	9 ^g	0	0	0	4 ^h
Repressible (+)	0	3 ⁱ	0	0	0	0	2 ^j
Repressible (-)	0	1 ^k	9 ^l	0	0	0	1 ^m

Wall, Hlavacek & Savageau. Nat Rev Genet 2004
<http://ecotfs.lanl.gov>

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Summary of Patterns in EcoTFs

Project

- Negative Self-Regulation
 - Preference observed in natural systems (33/49)
- Positive Self-regulation
 - All Inducible (+), D
 - Different functions and performance criteria
- Inverse coupling is not found among repressible systems
- Direct coupling is preferred for systems under repressor control

<http://ecotfs.lanl.gov>

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Patterns in Gene Regulation Coupling of Gene Expression

Wall, Hlavacek, & Savageau. JMB 2003

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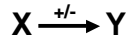
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Understanding Patterns in Natural Systems Search for Design Principles

Are natural designs optimal given ecological context?

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Demand Theory

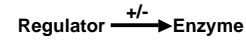


- + indicates positive mode of control, or activator control
- Indicates negative mode of control, or repressor control

Are there natural preferences for mode of control?

Demand Theory

Demand Theory, cont'd



Demand Theory:

If enzyme is in high demand, expect positive control

If enzyme is in low demand, expect negative control

Example of a design principle

Demand Theory, cont'd

High Demand



Loss of enzyme
High Penalty



Overexpression
Low penalty

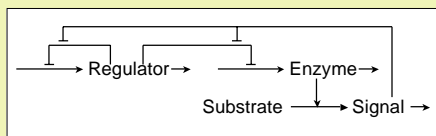
Any observed preference should be for positive control

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Models of Genetic Regulatory Circuits

Given knowledge of gene-circuit structure, what is gene-circuit function?



Does the level of enzyme go up or down with signal?

Need mathematical models

THE CONTROL OF THE FORMATION OF SPECIFIC PROTEINS IN BACTERIA AND IN ANIMAL CELLS
By LEO SZILARD*

THE ENRICO FERMI INSTITUTE FOR NUCLEAR STUDIES, THE UNIVERSITY OF CHICAGO
Communicated January 13, 1960

Szilard, PNAS 1960

Models of Gene Regulation Coupled ODEs

Boolean: Cynthia Reichardt Olson
Stochastic: Other session

Transcription rate

$$dX/dt = V^{(+)} - V^{(-)}$$

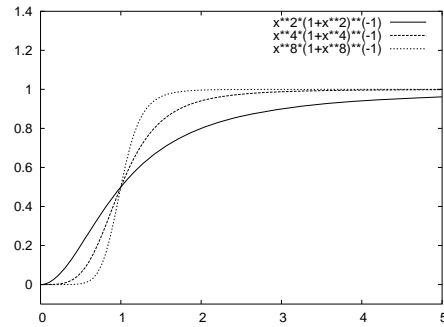
First-order decay/dilution

$$V^{(-)} = bX$$

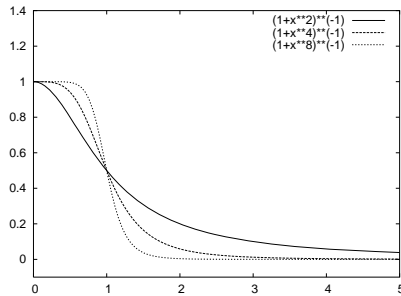
Hill-function regulation model

$$V^{(+)} = a(1 + R^n)^{-1} \quad \begin{array}{l} n < 0 \text{ activation} \\ n > 0 \text{ repression} \end{array}$$

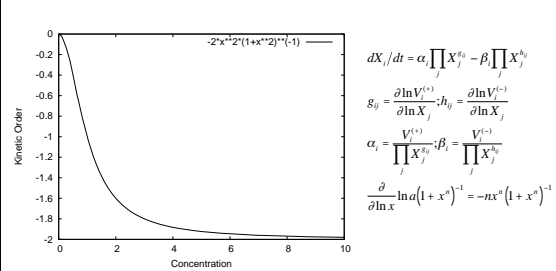
Model of Activation



Model of Repression



Power-Law Approximation



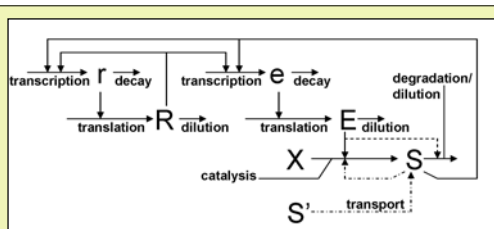
$$dX_i/dt = \alpha_i \prod_j X_j^{g_j} - \beta_i \prod_j X_j^{h_j}$$

$$g_j = \frac{\partial \ln V_i^{(+)}}{\partial \ln X_j}, h_j = \frac{\partial \ln V_i^{(-)}}{\partial \ln X_j}$$

$$\alpha_i = \frac{V_i^{(+)}}{\prod_j X_j^{g_j}}, \beta_i = \frac{V_i^{(-)}}{\prod_j X_j^{h_j}}$$

$$\frac{\partial}{\partial \ln x} \ln a(1+x^n)^{-1} = -nx^n(1+x^n)^{-1}$$

Elementary Gene Circuit Model



Dashes = inducible-catabolic gene circuit

Dot-dashes = repressible-biosynthetic gene circuit

Hlavacek & Savageau. JMB 1996; Wall, Hlavacek & Savageau. JMB 2003

Elementary Gene Circuit

Power-Law Mathematical Model

$$dX_1/dt = \alpha_1 X_6^{g_{16}} X_3^{g_{13}} X_5^{g_{15}} - \beta_1 X_1^{h_{11}}$$

$$dX_2/dt = \alpha_2 X_7^{g_{27}} X_1^{g_{21}} - \beta_2 X_2^{h_{22}}$$

$$dX_3/dt = \alpha_3 X_8^{g_{38}} X_9^{g_{39}} X_2^{g_{32}} X_3^{g_{33}} - \beta_3 X_2^{h_{32}} X_3^{h_{33}}$$

$$dX_4/dt = \alpha_4 X_6^{g_{46}} X_3^{g_{43}} X_5^{g_{45}} - \beta_4 X_4^{h_{44}}$$

$$dX_5/dt = \alpha_5 X_7^{g_{57}} X_4^{g_{54}} - \beta_5 X_5^{h_{55}}$$

X_1 = Enzyme mRNA

X_6 = nucleic acid pool

X_2 = Enzyme

X_7 = amino acid pool

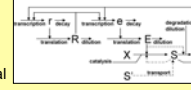
X_3 = Intracellular signal

X_8 = Signal precursor

X_4 = TF mRNA

X_9 = Extracellular signal

X_5 = TF



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- **Controlled mathematical comparisons**
- Structure-function relations

Controlled Mathematical Comparisons

- Compare performance criteria for alternative systems that carry out the same function (e.g. repressible systems)
- Internal Equivalence
 - Maintain identical parameter values for all processes but those being considered as alternative designs
 - All processes but transcription of regulator and enzyme are equivalent
- External Equivalence
 - Ensure that systems being compared have the same overall biological function
 - Steady-state effector gain is the same for alternative systems

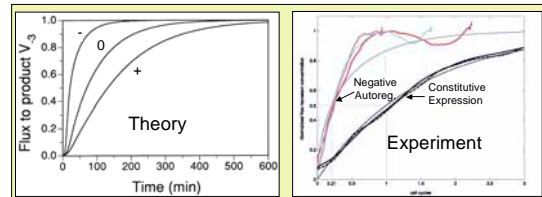
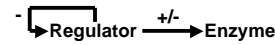
Savageau 1976 "Biochemical Systems Theory"

Performance Criteria for Elementary Gene Circuits

- Stability
 - Defined for stable systems as the ability of the steady-state enzyme level to remain stable, even when model parameter values may vary
 - Linearize equations about the steady-state solution, determine regions of parameter space with negative eigenvalues
 - Measure a distance in parameter space between a stable system and the nearest unstable system
- Steady-state robustness
 - Ability of the steady-state enzyme level to be maintained when model parameter values vary
 - Parameter sensitivity analysis
- Temporal responsiveness
 - Ability of the system to equilibrate quickly after a change in signal
 - Solve for system dynamics and calculate rise-times, decay-times, and settling times
- Information processing performance?
 - Mutual information
 - Predictive information

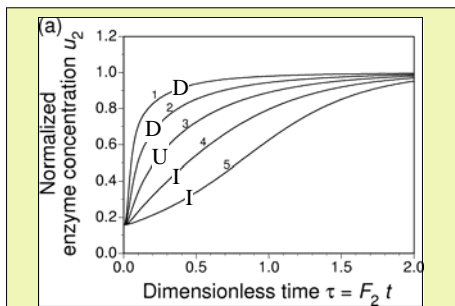
Project?

TF Negative Self-Regulation Increases Performance



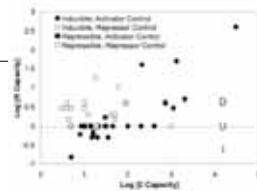
Savageau Nature 1974, 1975 Responsiveness: Rosenfeld, Elowitz & Alon. JMB 2002
 Hlavacek & Savageau. JMB 1995, 1996, 1997 Stability: Becskei & Serrano. Nature 2000
 Wall, Hlavacek & Savageau. JMB 2003 Robustness: Little et al. EMBO J 1999

Temporal Responsiveness Changes with Coupling Type



Prediction of Optimal Coupling Types

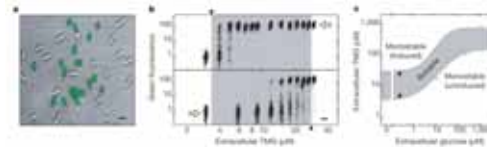
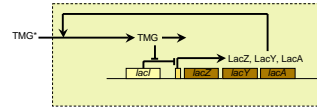
Effector TU	Gain		
	Low Gain	Intermediate	High Gain
Inducible (+)	Inverse	Uncoupled	Direct
Inducible (-)	Direct	Uncoupled	Inverse
Repressible (+)	Inverse	Direct	Direct
Repressible (-)	Direct	Uncoupled	Inverse



Contents

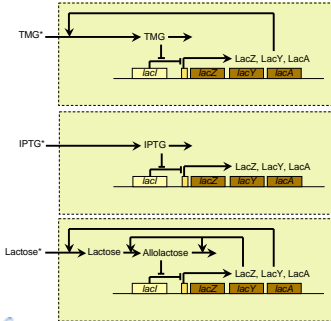
- Gene regulation
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Example: Bistability in *lac*



Ozbudak et al. Nature 2004

Importance of Biological Context

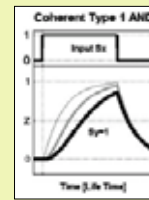
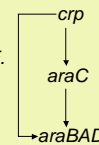


Project

- Is lac bistable in response to lactose?
- Novick " & Weiner, PNAS 1957
 - lac is bistable...
 - ...in response to TMG!

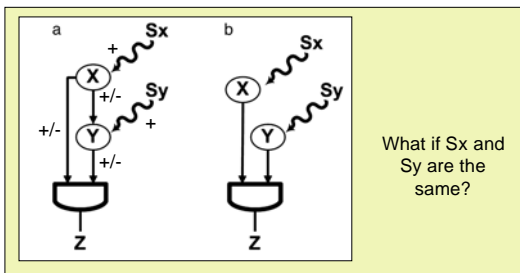
Structure-Function Relations Feed-Forward Loop

A network motif in the *E. coli* genetic regulatory network



Shen-Orr et al Nat Genet 2002
Mangan & Alon. PNAS 2003

Alternative Signal Interactions in FFLs

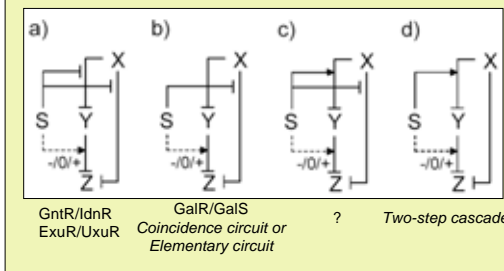


What if S_x and S_y are the same?

Mangan, S. and Alon, U. (2003) Proc. Natl. Acad. Sci. USA 100, 11980-11985

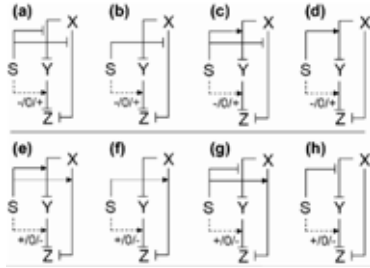
Type 2 Incoherent FFL

- Previous prediction of function: Repressible, Sign-sensitive accelerator



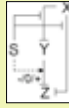
Analysis of an Exhaustive Set of Type 2 iFFLs

- a) (-,-,*)
 - *gntRKU-
idnDOTR-
gntKU*
 - *exuR-uxuR-
uxuAB*
- b) (0,-,*)
 - *galR-galS-
galETKM*



Symmetry between top & bottom systems

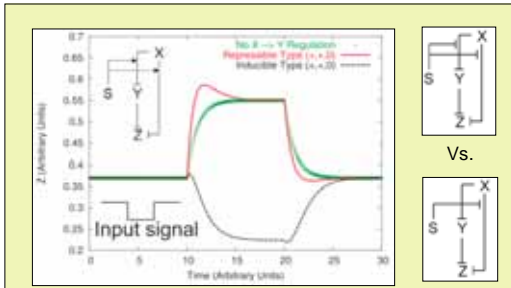
Mathematical Model of the Type 2 iFFL



$$H_{ij}(S_{ij}x_j/K_{ij}) = \left[1 + (S_{ij}x_j/K_{ij})^{n_{ij}} \right]^{-1}$$

$$\begin{aligned} dY/dt &= B_Y + \alpha_Y H'_{YX}(S_{YX}X/K_{YX}) - \beta_Y Y \\ dZ/dt &= B_Z + \alpha_Z H'_{ZX}(S_{ZX}X/K_{ZX}) H'_{ZY}(S_{ZY}Y/K_{ZY}) - \beta_Z Z \\ \beta_Y Y_{\infty} &= B_Y + \alpha_Y H'_{YX}(S_{YX}X/K_{YX}) \\ \beta_Z Z_{\infty} &= B_Z + \alpha_Z H'_{ZX}(S_{ZX}X/K_{ZX}) H'_{ZY}(S_{ZY}Y_{\infty}/K_{ZY}) \end{aligned}$$

Functions of the (+,+0) Type 2 iFFL



Performance of the (+,+0) Type 2 iFFL

- De-repression rise-time
 - Time required for the level of Z to come within 5% of its steady-state value
- De-repression settling-time
 - Time beyond which the level of Z never strays farther than 5% from its steady-state value
- Repression decay-time
- Repression settling-time

Equivalence Conditions for Comparisons

$$\begin{aligned} B_Z + \alpha_Z R'_{Z,\min} &= B'_Z + \alpha'_Z R'_{Z,\min} \\ B_Z + \alpha_Z R'_{Z,\max} &= B'_Z + \alpha'_Z R'_{Z,\max} \\ \alpha_Z &= \alpha'_Z \left(\frac{R'_{Z,\max} - R'_{Z,\min}}{R_{Z,\max} - R_{Z,\min}} \right) \\ B_Z &= B'_Z + \alpha'_Z R'_{Z,\min} - \alpha_Z R_{Z,\min} \end{aligned}$$

Equivalent steady-state Z at low and high signal

Parameter Sampling

- Three Hill coefficients n_{ij} : sample integers in range [1,8]
- Three thresholds K_{ij} : sample 11 log-randomly in range [0.01,100]
- 681,472 combinations in total
- Reference system:
 - $B_X = B_Y = 0.1$; $\alpha_X = \alpha_Y = 0.9$; $\beta_X = \beta_Y = \beta_Z = 1$, $K_{YX} = 1$, $K_{ZX} = 1$, $K_{ZY} = 1$; $n_{YX} = n_{ZX} = |n_{ZY}| = 2$, $X = 1$

Summary of (+,+,0) Type 2 μ FFL Temporal Responsiveness

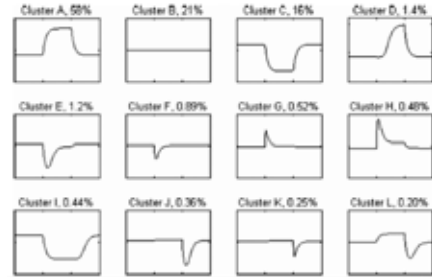
	Act 1:0			Act 2:0		
	Fastest	Slowest	Mean	Fastest	Slowest	Mean
Onset-to-peak time	0.08	0.46	0.26	0.26	0.44	0.35
Onset-to-settle time	0.12	0.46	0.42	0.23	0.44	0.33
Repression decay time	0.18	0.82	0.50	0.25	0.80	0.50
Repression settling time	0.11	0.82	0.50	0.24	0.80	0.50

Each entry is the fraction of 10,472 parameter combinations (see Methods) that cause the TF to have responsiveness measures that are faster than, the same as, or slower than equivalent circuits without X=+Y regulation. Two responsiveness measures are considered to be similar if their ratio equals 2.2:0.5.

- Derepression is either faster (rise) or slower (settle)
- Repression is faster

Clustering of Functional Responses

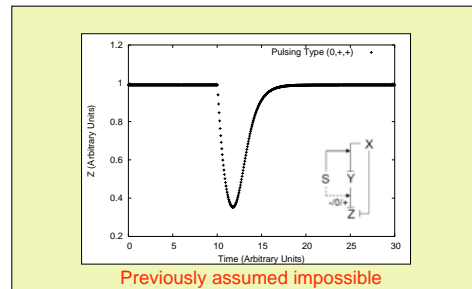
Project



Distribution of Functional Responses

Effect of Signal on TF Activities		% of Time Courses in Clusters A-L											Entropy		
ZX	YX	ZY	A	B	C	D	E	F	G	H	I	J		K	L
-	-	+	85	5	0	0	0	0	0	0	0	4	3	2	0.89
-	-	0	46	22	20	0	0	0	3	3	5	0	0	0	2.04
-	-	+	38	9	47	0	0	0	3	3	0	0	0	0	1.70
-	0	-	93	7	0	0	0	0	0	0	0	0	0	0	0.37
-	0	0	69	31	0	0	0	0	0	0	0	0	0	0	0.90
-	0	+	49	11	40	0	0	0	0	0	0	0	0	0	1.38
-	+	-	95	5	0	0	0	0	0	0	0	0	0	0	0.27
-	+	0	71	21	0	8	0	0	0	0	0	0	0	0	1.11
-	+	+	43	9	36	4	4	4	0	0	0	0	0	0	1.95
0	+	-	66	34	0	0	0	0	0	0	0	0	0	0	0.92
0	+	0	38	57	0	5	0	0	0	0	0	0	0	0	1.22
0	+	+	0	37	47	0	10	6	0	0	0	0	0	0	1.62
All			58	21	16	1	1	1	1	0	0	0	0	0	1.76

Two-Step Cascade Pulse Generator



Implications for Studies of Genetic Regulatory Networks

- The functions of binary gene circuits depend on signal interactions as well as genetic regulatory interactions
 - Inducible vs repressible
 - Intermediate steady-state response
 - Acceleration or delay of dynamics
 - Overshoot, undershoot, or pulsing
- Genome-wide characterization of genetic regulatory networks will require knowledge of both **signal interactions** and **genetic regulatory interactions** (EcoTFs)

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