Non-equilibrium relaxation in the vicinity of the extinction critical point in a stochastic lattice Lotka-Volterra model

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Short Abstract — **Spatially extended stochastic models for predator-prey competition and coexistence display complex, correlated spatio-temporal structures and are governed by large fluctuations. We specifically study a stochastic lattice Lotka-Volterra model. Generically, the system tends to relax into a quasi-stationary state, independent of the initial conditions. We investigate the non-equilibrium relaxation in the vicinity of the critical point. We obtain a power law dependence between the relaxation time and predation rate (critical slowing down), and measure the critical aging scaling exponents.**

Keywords **— non-equilibrium relaxation, stochastic lattice Lotka-Volterra model, critical point, critical exponent.**

I. Background

The Lotka-Volterra model [1,2] describes a two-species predator-prey coexistence / competition system with 'predators' A and 'prey' B. Predators spontaneously die with rate μ. They also may consume prey and reproduce with rate λ > 0. Prey may reproduce with rate σ.

$$A \xrightarrow{\mu} 0$$
$$AB \xrightarrow{\lambda} AA$$
$$B0 \xrightarrow{\sigma} BB$$

We study a stochastic lattice Lotka-Volterra model [3,4] by means of Monte Carlo simulations performed on a 2D square lattice with periodic boundary conditions. Site restrictions are applied to the system. Each lattice site can either be empty, occupied by a 'predator' or by a 'prey'.

If we fix μ and σ, the system relaxes into one of two possible states which are governed by predation rate λ. There is one stable state in which both species survive. The other state is an absorbing state with only prey remaining. A critical predator extinction threshold exists between these two states.

II. Summary of the Results

We set the initial condition as a random configuration with both species densities 0.3 on a 1024*1024 lattice. The system relaxes into a quasi-stationary state with reaction rates $\sigma = 1$, $\mu = 0.025$ and $\lambda_1 = 0.25$. Then we change the predation rate from λ_1 to λ_2, followed by an ensuing relaxation to another state.

A. Relaxation between two quasi-stationary states

We obtain non-equilibrium relaxation between two quasi-stationary states when λ_2 is in the stable region. The density relaxes into a new state exponentially. The ensuing relaxation times are measured via the peak width of the population density Fourier transforms. The damping rate is equal to the inverse of the relaxation time.

Away from the critical point, we find that the initial configuration influences the oscillations for the duration of one relaxation time.

B. Quench to the critical point

In the vicinity of the critical point λ_c, we obtain a power law dependence of the relaxation time on $(\lambda-\lambda_c)/\lambda_c$ (critical slowing down). The associated dynamical critical exponent is measured to be $z\upsilon \approx 1.9$ for a 512*512 system.

We employ different system sizes to carry out finite-size scaling [5] in order to accurately measure the aging scaling exponents.

III. Conclusion

In the lattice Lotka-Volterra Model, there is a species coexistence state where both predators and prey can stably survive. We observe relaxation between two quasi-stable states when changing the predation rate. As expected, we find that the initial state generically only influences the oscillations for the duration of about one relaxation time, implying that the system quickly loses any memory of the initial configuration.We have measured the critical aging scaling exponents following a quench of the system to the predator extinction threshold.

References

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Acknowledgements: This work was funded by U. S. Department of Energy, Office of Basic Energy Sciences, Division of Materials Science and Engineering under Award DEFG02-09ER46613

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