Estimation of multifractal properties of random walks observed through finite probing windows

G. M. Viswanathan[1,2], L. Giuggioli[1,3], and V. M. Kenkre[1]

***Short Abstract* — Random walks have fractal properties in the sense that a rescaled random walk has statistical properties very similar to that of the original, and appear qualitatively the same on any time scale. Many random walks have a mean square displacement that grows linearly in time. More generally, the $2n$-th moment of the displacement scales as a power of time $t^{2nH(n)}$, where the scaling exponent H is known as the Hurst exponent. Monofractal walks have unique exponents $H(n)=H$, whereas multifractal walks have non-unique $H(n)$. There are a number of biological instances in which finite sized probing windows need to be considered, from examples of cells and bacteria in confined spaces to rodent populations of interest in epidemic spread. A monofractal random walk seen through a finite probing window will incorrectly appear to have multifractal properties [1]. We propose a method to correct for such spurious detection of multifractality. We then study the case in which the roaming region of the walker is itself of limited extent, so that a nonlinear interplay occurs between the roaming area and the window size.**

***Keywords* — Random walks, epidemic spread, multifractals, finite probing windows.**

I. Purpose

THE study of random walks began with the quantitative description of Brownian motion. The square root of the second moment of the probability density function of the displacement of Brownian walkers grows as the square root of time. The $2n$-th root of the $2n$-th moments grow in time in the same manner, so that a single power law scaling exponent $H=1/2$ quantifies the whole process. Such behavior is known as monofractal scaling and H is known as the Hurst exponent. In contrast, if each moment grows with time with its own unique exponent, the result is multifractal scaling.

In this contribution, we show analytically that monofractal motion may incorrectly appear to have multifractal properties if the measuring apparatus has a finite window of observation. We discuss a number of aspects related to this issue and propose a method to correct for this artifact. Finally, we consider the case in which the random walker remains confined to a finite region. In this case, not only are the measurements confined to the probing window, but the walkers themselves are confined, leading to a nonlinear interplay between the two confining effects.

II. Conclusion

In summary, we show that finite probing windows may lead to the spurious detection of multifractality and have proposed a method to correct for such spurious results. We conclude that special care must be taken when investigating the multifractal properties of random walks seen through a finite probing window.

References

[1] Giuggioli L, Viswanathan GM, Kenkre VM, Parmenter RR, and Yates TL (2007). Effects of Finite Probing Windows on the Interpretation of the Multifractal Properties of Random Walks. *EPL* **77,** 40004.

Acknowledgements: This work was funded by NIH grant XX00000. This footnote is optional.

[1]Consortium of the Americas for Interdisciplinary Science and Department of Physics and Astronomy, University of New Mexico, Albuquerque NM 87131, USA.

[2]Permanent address: Instituto de Física, Universidade Federal de Alagoas, CEP 57072-970, Maceió-AL, Brazil. E-mail: Gandhi.Viswanathan@pq.cnpq.br

[3]Present address: Princeton Environmental Institution, Princeton University, Princeton, NJ 08544-1003, USA. E-mail: lgiuggio@Princeton.EDU