Universality of Poisson indicator and Fano factor of transport event statistics in ion channels

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Short Abstract — **We consider a generic stochastic model of ion transport through a channel with arbitrary internal structure and kinetic rates of transitions between internal states. We show that measurement of statistics of single molecule transition time through the channel contains only restricted information about internal structure of the channel. In particular, the most accessible flux fluctuation characteristics, such as the Poisson Indicator (P) and the Fano Factor (F) as function of solute concentration, depend only on three parameters in addition to the parameters of the Michaelis-Menten curve that characterizes average current through the channel. Nevertheless, measurement of Poisson indicator or Fano factor for such renewal processes can discriminate reactions with multiple intermediate steps as well as provide valuable information about the internal kinetic rates**

I. INTRODUCTION

The key measurable quantity in ion channel transport had been the steady state flux through a single channel in a membrane that separates two compartments with different solute concentrations. MM law was found to describe the transport of solute molecules through a class of ion channels, in which J becomes the ion flux through the channel and [S] is the solute concentration on one side of the channel, assuming that [S]=0 on the other side. Measurement of average flux is intrinsically limited as only two constants can be obtained experimentally by measuring MM-curve. The advance of single molecule techniques allowed researchers to alleviate this restriction by studying not only average currents but also the *statistics* of single molecule transitions in the channel-facilitated transport through biological membranes[1,2]. The most accessible characteristics of fluctuations in molecular transport are related to second moment of the turnover time statistics and current distribution such as the Poisson indicator(P) and the Fano Factor(F). We consider a generic stochastic model of ion transport through a channel with arbitrary internal structure and kinetic rates of transitions between internal states. We show that, similarly to the universality of average flux characteristics, the complexity of P and F for transport through ion channels reduces to the universal functions that depend on, maximum, three additional constant parameters. We also show that one can derive a connection between the Fano factor and Poisson indicator for such renewal kinetic processes.

II. RESULTS AND CONCLUSION

The central object of our theory is to calculate the first passage time distribution between two successive monitored events[3]. For example, in ion channels, monitored transitions can be events when transported molecule leaves the ion channel. By considering higher moments of the turnover time distribution, we calculate the Poisson indicator, P. The parameter P is, generally, a non-monotonous function of the solute concentration [S], and at high solute concentrations the statistics of turnover time distribution is non-Poisson. Using the relation between the turnover time probability distribution and the cumulant generating function[4], we express the Fano factor[5] in terms of derivatives of the turnover time distribution and obtain a similar universality to the Poisson indicator.

In this work, we showed that the Poisson indicator and the Fano factor have simple generic functional dependences on solute concentration irrespective of the number of internal states in the ion channel kinetic model. We also show that kinetic models with more than two internal states may still be distinguishable from 2-state models if variances of fluxes are measured.

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Acknowledgements: This work was funded by NIH grant ECCS-0925618

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