Homework assignment for Q-bio Summer School, LANL, 2008

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Problem 1. Consider left and right eigenvectors $\mathbf{u_L}$ and $\mathbf{u_R}$ of a nonsymmetric matrix H with nonzero determinant, and corresponding eigenvalues λ_L and λ_R.

$$\mathbf{u_L} H = \lambda_L \mathbf{u_L}, \quad H \mathbf{u_R} = \lambda_R \mathbf{u_R} \tag{1}$$

(a) prove that the sets of possible left and right eigenvalues of matrix H coincide, i.e. for every nonzero eigenvalue λ_R there is $\lambda_L = \lambda_R$.

(b) let $\mathbf{u_L}$ correspond to eigenvalue λ_L and $\mathbf{u_R}$ corresponds to eigenvalue λ_R, and $\lambda_R \neq \lambda_L$. Show that the scalar product of these eigenvectors is zero, i.e. $\mathbf{u_L} \cdot \mathbf{u_R} = 0$.

(c) consider arbitrary n-dimensional vector. Show that it can be written in the basis of right eigenvectors of an $n \times n$ matrix H.

(d) what are coefficients of this decomposition in terms of elements of left and right eigenvectors of H?

Problem 2. This problem has not been considered in existing literature, however, its resolution can be important for understanding the shot noise in an important biochemical process. The answer to it is unknown but it is clear that its analytical solution is accessible.

Consider a mechanism of enzyme mediated reaction

$$S + E \underset{k_{-1}[SE]}{\overset{k_1[S]}{\rightleftharpoons}} SE \underset{k_{-3}[SP]}{\overset{k_3[SE]}{\rightleftharpoons}} SP \xrightarrow{k_2[SP]} E + P. \tag{2}$$

Unlike a standard Michaelis-Menten mechanism, here we take into account an extra step in order to include conformational changes of SE complex into a structurally different SP conformation.

By analogy with the MM enzyme, derive average rate of conversion of S into P at large time scales. Possible steps may include

1. Write the master equations for probabilities of states of enzyme to be either free, in SE, or in SP states. Determine the corresponding evolution matrix H_0 and its left and right eigenstates,

corresponding to zero eigenvalue. This can be done analytically. MATHEMATICA provides all needed routines to speed up these calculations.

2. Knowing H_0 determine the evolution matrix $H(\chi)$ simply by marking a proper off-diagonal matrix element by $e^{i\chi}$.

3. The main complication in comparison with simplest MM-reaction comes from the fact that now the matrix $H(\chi)$ has dimensions 3×3, and thus its eigenvalues have very complicated form.

4. Instead of finding the whole eigenvalue $\lambda_0(\chi)$ of $H(\chi)$ with the lowest real part, note that to extract first cumulants, it is enough to treat parameter χ as small. Thus to find 2nd cumulant it is sufficient to find $\lambda_0(\chi)$ only up to $O(\chi^2)$. For this write $H(\chi)$ in the form

$$H(\chi) = H_0 + V(\chi), \tag{3}$$

assuming that $V(\chi)$ is a small correction.

5. Find the perturbative solution for $\lambda_0(\chi) \approx iI_1\chi - I_2\chi^2/2$. It should be helpful to work by analogy with similar quantum mechanical perturbation theory, presented e.g. in Ref.[1].

6. Find the Fano factor. Explore this solution in limiting cases, relevant to many applications, such as $k_{-1} \gg k_2$. Compare result with that of standard MM-kinetics.

7. Write an article and submit it for publication.

[1] David J. Griffiths, *Introductory Quantum Mechanics*, Benjamin Cummings; 2 edition (April 10, 2004)

[2] N. A. Sinitsyn and I. Nemenman, *EPL* **77**, 58001 (2007)

[3] D. A. Bagrets and Y. V. Nazarov, *Phys. Rev. B* **67**, 085316 (2003).

[4] L. Michaelis and M. L. Menten, *Biochem. Z.* **49**, 333 (1913).