Cancer dynamics & Game Theory: An espacially extended model

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1 Short Abstract

We propose a modified version of replicator dynamics model for capture the exchange of chemical signals (in a payoff matrix), likewise simulate the interactions and competition between normal and multiple myeloma cells, incluiding explicitly aspects of the spatial structure of the phenomenon.

Key words: Evolutionary Game Theory, Replicator Equation, Cancer.

2 Replicator dynamics and Spatially Extended Model

The development of cancer is associated with the expansion of a mutant clone, however, because to the interactions with other cells is best described in terms of a frequency-dependent selection (FDS) [1]. The replicator equation (eq.1) surged in Evolutionary Game Theory and has been used in many fields as population genetics, evolutionary dynamics, chemical network [2] and cancer. This equation describes evolutionary game dynamics i.e. FDS, among n different phenotypes in the deterministic limit of an infinitely large, whithout mutations and well-mixed population, for this reason is an appropriate mathematical approach in this studies.

$$\frac{dx_i}{dt}(t) = x_i(t)[f(\vec{x}) - \phi(\vec{x})] \tag{1}$$

where x_i= frecuency of i type, $f(\vec{x}) = \sum_{j=1}^{n} a_{ij}x_j$ is the fitness of i type and $\phi(\vec{x}) = \sum_{i=1}^{n} f_i(\vec{x})x_i$ is the averange fitness in the population [3]. One weak point in this equation is that it does not consider structured populations. Then we trace a mesh where the neighborhood is given by $(C_{ij}) = C_{kl}/|k-i|/+|l-j| \leq 1 = \{C_{i-1,j}, C_{i,j-1}, C_{i,j}, C_{i,j+1}, C_{i-1,j}\}$ for cyclic boundary conditions and locate the cellular types into the mesh. Explicitly,

$$\frac{dx_i^{ij}}{dt}(t) = x_i^{ij}(t)[f_i^{ij}(\vec{x}) - \Phi^{ij}(\vec{x})] \tag{2}$$

Where $\Phi^{ij} = \frac{1}{5}\sum_1^5 f_{vi}$ is the parameter that spacially couples the populations in the whole mesh. We used this formulation for modelling the invasion of multiple myeloma cells into the bone, because the exchange of chemical signals is favourable for this phenotypes and analyze the dynamic behavior in one by one the cells in this mesh or bone.

3 Conclusions

We observated that the spatial population dynamics without coupling does not reflect the experimental results, while if he does coupled dynamics and the stability of this system depends on the parameters of interaction between the cell types. We present the model and analitical results.

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