Rule-based Modeling

Bill Hlavacek

Theoretical Division Los Alamos National Laboratory







Who is this?



http://bionetgen.org

Cellular regulatory systems are complex

									and a second sec					
		GRIMS		PRKAF	R1A	The state of the s	and an and a state	Contraction of the second	and Saturdant					
Home	Brows	e	FAQ's				Immune Si	gnaling Pa	athways		Cancer Si	gnaling Pat	hways:	
	eneline P	athursu							Accession n	mbor i Noti	ath 4			
GFRIS	gnaling P	atnway							Accession in	inder i Neu	raun_4			
ne epider GFR/erbB	family of	n factor re receptor ty	ceptor (EGI /rosine kina	-K1) is a co ises. Upon	binding to	receptor t its ligand	nat belongs Epidermal (fo the Growth	Pathway S	tatistics				
actor (EC	GF), this	receptor ca	n undergo	homodime	rization or	heterodi	merize with	other	Molecules I	ivolved				177
ubunits. 1	n addition	to EGF, th	e EGFR1 al	so binds to	transform	ng growth	ion of the re i factor alph	a (TGF	Enzyme Cat	eractions				67
pha), Am	phiregulin,	Betacelluli	n, Epireguli	n and Hepa	rin-binding	EGF-like g	rowth facto	or (HB-	Transport	a1y515				28
ar). Ligai		ictivates the		IAF K Signan	ing modules				Genes Tran	scriptionally	Regulated			285
										(Pathway	Authority		
				(* -4-1	(77)					(Pathway Commen	Authority ts		
Molecules	Involved in	n EGFR1 sig AP2A1	naling pathv APPL1	vay (Total = APPL2	177) ARAF	ARF4	ATF1	BCAR1	CAMK2A	((Commen	Authority) ts	CBL	CBLB
Molecules <u>ABI1</u> CBLC	S Involved in <u>AKT1</u> CDC42	n EGFR1 sig <u>AP2A1</u> CEACAM1	naling pathv <u>APPL1</u> CEBPA	vay (Total = <u>APPL2</u> CEBPB	177) <u>ARAF</u> CREB1	ARF4 CRK	ATF1 CRKL	BCAR1 CSK	CAMK2A CTNND1	((CASP9 DDEF1	Commen Commen CAV1 DNM1	Authority ts <u>CAV2</u> DOK2	<u>CBL</u> DUSP1	CBLB EEF1A1
Molecules ABI1 CBLC GF	EINVOIVED in <u>AKT1</u> <u>CDC42</u> EGFR	n EGFR1 sig <u>AP2A1</u> <u>CEACAM1</u> ELF3	naling pathv <u>APPL1</u> <u>CEBPA</u> ELK1	vay (Total = <u>APPL2</u> <u>CEBPB</u> ELK4	177) ARAF <u>CREB1</u> EPN1	ARF4 <u>CRK</u> EPPK1	ATF1 CRKL EPS15	BCAR1 CSK EPS15L:	CAMK2A CTNND1 L EPS8	CASP9 DDEF1 ERRFI1	Pathway Commen CAV1 DNM1 FOS	Authority ts CAV2 DOK2 FOX01A	CBL DUSP1 GAB1	CBLB EEF1A1 GAB2
Molecules ABI1 CBLC GF GIT1	i Involved ii <u>AKT1</u> <u>CDC42</u> <u>EGFR</u> GJA1	n EGFR1 sig <u>AP2A1</u> <u>CEACAM1</u> <u>ELF3</u> GRB10	naling pathv <u>APPL1</u> <u>CEBPA</u> <u>ELK1</u> GRB14	vay (Total = <u>APPL2</u> <u>CEBPB</u> <u>ELK4</u> GRB2	177) ARAF <u>CREB1</u> EPN1 GRB7	ARF4 CRK EPPK1 HAT1	ATF1 CRKL EPS15 HD	BCAR1 CSK EPS15L: HDAC1	CAMK2A CTNND1 EP58 HIP1	(CASP9 DDEF1 ERRFI1 HIST3H3	Cavi DNM1 FOS HRAS	Authority ts CAV2 DOK2 FOX01A INPPL1	CBL DUSP1 GAB1 ITCH	CBLB EEF1A1 GAB2 JAK1
Molecules ABI1 CBLC GF GIT1 AK2	Involved in AKT1 CDC42 EGFR GJA1 JUN	n EGFR1 sig AP2A1 CEACAM1 ELF3 GRB10 JUND	naling pathv APPL1 CEBPA ELK1 GRB14 KLF11	vay (Total = <u>APPL2</u> <u>CEBPB</u> <u>ELK4</u> <u>GRB2</u> <u>KRAS</u>	177) ARAF <u>CREB1</u> <u>EPN1</u> <u>GRB7</u> <u>KRT17</u>	ARF4 CRK EPPK1 HAT1 KRT18	ATF1 CRKL EPS15 HD KRT7	BCAR1 CSK EPS15L HDAC1 KRT8	CAMK2A CTNND1 EP58 HIP1 MAP2K1	CASP9 DDEF1 ERRFI1 HIST3H3 MAP2K2	CAV1 Commen CAV1 DNM1 FOS HRAS MAP2K3	Authority) ts CAV2 DOK2 FOXO1A INPPL1 MAP2K5	CBL DUSP1 GAB1 ITCH MAP2K7	CBLB EEF1A1 GAB2 JAK1 MAP3K1
Molecules ABI1 CBLC GF GIT1 AK2 MAP3K14	Involved in AKT1 CDC42 EGFR GJA1 JUN MAP3K2	n EGFR1 sig AP2A1 CEACAM1 ELF3 GRB10 JUND MAP3K3	naling pathy APPL1 CEBPA ELK1 GRB14 KLF11 MAP3K4	vay (Total = <u>APPL2</u> <u>CEBPB</u> <u>ELK4</u> <u>GRB2</u> <u>KRAS</u> MAPK1	177) ARAF CREB1 EPN1 GRB7 KRT17 MAPK14	ARF4 CRK EPPK1 HAT1 KRT18 MAPK3	ATF1 CRKL EPS15 HD KRTZ MAPKZ	BCAR1 CSK EPS15L: HDAC1 KRT8 MAPK8	CAMK2A CTNND1 EP58 HIP1 MAP2K1 MCF2	CASP9 DDEF1 ERRF11 HIST3H3 MAP2K2 MTA2	CAV1 Commen CAV1 DNM1 FOS HRAS MAP2K3 MYC	Authority) ts CAV2 DOK2 FOX01A INPPL1 MAP2K5 NCK1	CBL DUSP1 GAB1 ITCH MAP2K7 NCK2	CBLB EEF1A1 GAB2 JAK1 MAP3K1 NDUFA13
Molecules ABI1 CBLC GF SIT1 AK2 MAP3K14 NRAS	Involved in AKT1 CDC42 EGFR GJA1 JUN MAP3K2 PAK1	n EGFR1 sig AP2A1 CEACAM1 ELF3 GRB10 JUND MAP3K3 PEBP1	naling pathy APPL1 CEBPA ELK1 GRB14 KLF11 MAP3K4 PIK3C2B	vay (Total = APPL2 CEBPB ELK4 GRB2 KRAS MAPK1 PIK3CA	177) ARAF CREB1 EPN1 GRB7 KRT17 MAPK14 PIK3CB	ARF4 CRK EPPK1 HAT1 KRT18 MAPK3 PIK3CD	ATF1 CRKL EPS15 HD KRT7 MAPK7 PIK3CG	BCAR1 CSK EP515L: HDAC1 KRT8 MAPK8 PIK3R1	CAMK2A CTNND1 EPS8 HIP1 MAP2K1 MCF2 PIK3R2	CASP9 DDEF1 ERRF11 HIST3H3 MAP2K2 MTA2 PIK3R3	CAV1 Commen CAV1 DNM1 FOS HRAS MAP2K3 MYC PITPNA	Authority) ts CAV2 DOK2 FOXO1A INPPL1 MAP2K5 NCK1 PKN2	CBL DUSP1 GAB1 ITCH MAP2K7 NCK2 PLCG1	CBLB EEF1A1 GAB2 JAK1 MAP3K1 NDUFA13 PLCG2
Molecules ABI1 CBLC CGF SIT1 IAK2 IAP3K14 NRAS PLD1	i Involved i AKT1 CDC42 EGFR GJA1 JUN MAP3K2 PAK1 PLD2	n EGFR1 sig AP2A1 CEACAM1 ELF3 GRB10 JUND MAP3K3 PEBP1 PLEC1	naling pathy APPL1 CEBPA ELK1 GRB14 KLF11 MAP3K4 PIK3C2B PLSCR1	vay (Total = APPL2 CEBPB ELK4 GRB2 KRAS MAPK1 PIK3CA PRKAR1A	ARAF CREB1 EPN1 GRB7 KRT17 MAPK14 PIK3CB PRKCA	ARF4 CRK EPPK1 HAT1 KRT18 MAPK3 PIK3CD PRKCB1	ATF1 CRKL EPS15 HD KRTZ MAPKZ PIK3CG PRKCG	BCAR1 CSK EPS15L: HDAC1 KRT8 MAPK8 PIK3R1 PRKCI	CAMK2A CTNND1 EPS8 HIP1 MAP2K1 MCF2 PIK3R2 PRKCZ	CASP9 DDEF1 ERRFI1 HIST3H3 MAP2K2 MTA2 PIK3R3 PRKD1	CAV1 Commen FOS HRAS MAP2K3 MYC PITPNA PTK2B	Authority) ts CAV2 DOK2 FOX01A INPPL1 MAP2K5 NCK1 PKN2 PTK6	CBL DUSP1 GAB1 ITCH MAP2K7 NCK2 PLCG1 PTPN11	CBLB EEF1A1 GAB2 JAK1 MAP3K1 NDUFA13 PLCG2 PTPN12
Molecules ABI1 CBLC CGF GIT1 IAK2 MAP3K14 NRAS PLD1 PTPN5	AKT1 CDC42 EGFR GJA1 JUN MAP3K2 PAK1 PLD2 PTPN6	n EGFR1 sig AP2A1 CEACAM1 ELF3 GRB10 JUND MAP3K3 PEBP1 PLEC1 PTPRR	naling pathy APPL1 CEBPA ELK1 GRB14 KLF11 MAP3K4 PIK3C2B PLSCR1 PXN	vay (Total = APPL2 CEBPB ELK4 GRB2 KRAS MAPK1 PIK3CA PRKAR1A RAB5A	ARAF CREB1 EPN1 GRB7 KRT17 MAPK14 PIK3CB PRKCA RAC1	ARF4 CRK EPPK1 HAT1 KRT18 MAPK3 PIK3CD PRKCB1 RAF1	ATF1 CRKL EPS15 HD KRTZ MAPKZ PIK3CG PRKCG RALB	BCAR1 CSK EPS15L: HDAC1 KRT8 MAPK8 PIK3R1 PRKCI RALBP1	CAMK2A CTNND1 EPS8 HIP1 MAP2K1 MCF2 PIK3R2 PRKCZ RALGDS	CASP9 DDEF1 ERRFI1 HIST3H3 MAP2K2 MTA2 PIK3R3 PRKD1 RASA1	CAV1 Commen CAV1 DNM1 FOS HRAS MAP2K3 MYC PITPNA PTK2B RBBP7	Authority) ts CAV2 DOK2 FOX01A INPPL1 MAP2K5 NCK1 PKN2 PTK6 REPS1	CBL DUSP1 GAB1 ITCH MAP2K7 NCK2 PLCG1 PTPN11 REPS2	CBLB EEF1A1 GAB2 JAK1 MAP3K1 NDUFA13 PLCG2 PTPN12 RFXANK
Molecules ABI1 CBLC CGF SIT1 IAK2 MAP3K14 NRAS PLD1 PTPN5 RG516	AKT1 CDC42 EGFR GJA1 JUN MAP3K2 PAK1 PLD2 PTPN6 RIPK1	n EGFR1 sig AP2A1 CEACAM1 ELF3 GRB10 JUND MAP3K3 PEBP1 PLEC1 PTPRR RPS6KA1	naling pathy APPL1 CEBPA ELK1 GRB14 KLF11 MAP3K4 PIK3C2B PLSCR1 PXN RPS6KA2	vay (Total = APPL2 CEBPB ELK4 GRB2 KRAS MAPK1 PIK3CA PRKAR1A RAB5A RPS6KA3	ARAF CREB1 EPN1 GRB7 KRT17 MAPK14 PIK3CB PRKCA RAC1 RPS6KA5	ARF4 CRK EPPK1 HAT1 KRT18 MAPK3 PIK3CD PRKCB1 RAF1 SH2D3C	ATF1 CRKL EPS15 HD KRTZ MAPKZ PIK3CG PRKCG RALB SH3BGRL	BCAR1 CSK EPS15L: HDAC1 KRT8 MAPK8 PIK3R1 PRKCI RALBP1 SH3GL2	CAMK2A CTNND1 EPS8 HIP1 MAP2K1 MCF2 PIK3R2 PRKCZ RALGDS SH3GL3	CASP9 DDEF1 ERRFI1 HIST3H3 MAP2K2 MTA2 PIK3R3 PRKD1 RASA1 SH3KBP1	Pathway Commen CAV1 DNM1 FOS HRAS MAP2K3 MYC PITPNA PTK2B RBBP7 SHC1	Authority) ts CAV2 DOK2 FOXO1A INPPL1 MAP2K5 NCK1 PKN2 PTK6 REPS1 SHOC2	CBL DUSP1 GAB1 ITCH MAP2K7 NCK2 PLCG1 PTPN11 REPS2 SIN3A	CBLB EEF1A1 GAB2 JAK1 MAP3K1 NDUFA13 PLCG2 PTPN12 RFXANK SMAD2
Molecules ABI1 CBLC CGF SIT1 IAK2 4AP3K14 NRAS PLD1 PTPN5 RGS16 SMAD3	AKT1 CDC42 EGFR GJA1 JUN MAP3K2 PAK1 PLD2 PTPN6 RIPK1 SNCA	n EGFR1 sig AP2A1 CEACAM1 ELF3 GRB10 JUND MAP3K3 PEBP1 PLEC1 PTPRR RPS6KA1 SNRPD2	naling pathy APPL1 CEBPA ELK1 GRB14 KLF11 MAP3K4 PIK3C2B PLSCR1 PXN RPS6KA2 SOCS1	vay (Total = APPL2 CEBPB ELK4 GRB2 KRAS MAPK1 PIK3CA PRKAR1A RAB5A RPS6KA3 SOCS3	ARAF CREB1 EPN1 GRB7 KRT17 MAPK14 PIK3CB PRKCA RAC1 RPS6KA5 SOS1	ARF4 CRK EPPK1 HAT1 KRT18 MAPK3 PIK3CD PRKCB1 RAF1 SH2D3C SOS2	ATF1 CRKL EPS15 HD KRT7 MAPK7 PIK3CG PRKCG RALB SH3BGRL SP1	BCAR1 CSK EPS15L: HDAC1 KRT8 MAPK8 PIK3R1 PRKCI RALBP1 SH3GL2 SPRY2	CAMK2A CTNND1 EPS8 HIP1 MAP2K1 MCF2 PIK3R2 PIK3R2 PRKCZ RALGDS SH3GL3 SRC	CASP9 DDEF1 ERRFI1 HIST3H3 MAP2K2 MTA2 PIK3R3 PRKD1 RASA1 SH3KBP1 STAT1	Pathway Commen CAV1 DNM1 FOS HRAS MAP2K3 MYC PITPNA PITPNA PTK2B RBBP7 SHC1 STAT2	Authority) ts CAV2 DOK2 FOX01A INPPL1 MAP2K5 NCK1 PKN2 PTK6 REPS1 SHOC2 STAT3	CBL DUSP1 GAB1 ITCH MAP2K7 NCK2 PLCG1 PTPN11 REPS2 SIN3A STAT5A	CBLB EEF1A1 GAB2 JAK1 MAP3K1 NDUFA13 PLCG2 PTPN12 RFXANK SMAD2 STAT5B

Akhilesh Pandey (Johns Hopkins)

Value added by modeling

- 1. We can use models to organize information about a system with precision
- 2. We can determine the logical consequences of a model specification

Outline

1. Combinatorial complexity

- 2. The conventional approach to modeling
- 3. The rule-based approach to modeling
- 4. Tools
- 5. New simulation methods

Signaling proteins contain domains and motifs that mediate interactions with other proteins





Epidermal growth factor receptor (EGFR) EGF ECD 9 sites \Rightarrow 2⁹=512 phosphorylation states тм PTK Y869 Y915 Src Y944 Y1016 PLC-y Abl Y1092 Grb2 Y1110 Y1125 Dok-R (PTB-1B) Y1172 Shc SHP-1 Y1197 EGFR





Signaling proteins typically contain multiple phosphorylation sites



Source: Phospho.ELM database v. 3.0 (http://phospho.elm.eu.org)

Oligomerization alone can generate many complexes

Complexes potentially involved in Toll-like receptor signaling



A hexamer of death domains Weber and Vincenz (2001) *FEBS Lett*.



Complexes of TIR domains C.-T. Tung (Los Alamos)

The problem of combinatorial complexity necessitates a new modeling approach

- Inside a Chemical Plant
 - Large numbers of molecules...
 - ... of a few types
 - Conventional modeling works fine
- Inside a Cell
 - Small numbers of molecules...
 - ...of many possible types
 - Rule-based modeling addresses this situation

The need for predictive models of large scale with site-specific details

- Molecular changes that affect cell signaling cause disease (cancer)
- Over 200 drugs that target malfunctioning signaling proteins are currently in clinical trials
 - One spectacular success (Gleevec)
 - But results are largely disappointing for most patients
- 96 clinical trials are underway to test combinations of drugs (clinicaltrials.gov)
 - There are too many combinations to consider all of them in trials

Outline

- 1. The biochemistry of cell signaling and combinatorial complexity
- 2. The conventional approach to modeling
- 3. The rule-based approach to modeling
- 4. Tools
- 5. New simulation methods

Models can be specified in different ways

Conventional representation of a biochemical reaction network

$$\begin{bmatrix} \bigcirc A \\ \swarrow & \bigcirc \\ X_{00} & X_{10} \\ 1 & \bigcirc \\ X_{00} & X_{10} \\ 1 & \bigcirc \\ X_{01} & X_{11} \\ X_{01} & X_{11} \\ X_{01} & X_{11} \\ X_{01} & X_{11} \\ \end{bmatrix} \begin{bmatrix} \bigcirc & \bigcirc \\ X_{00} & X_{10} \\ X_{10} & B \\ X_{01} & X_{11} \\ X_{01} & A \\ X_{01} & X_{11} \\ \end{bmatrix} \begin{bmatrix} \bigcirc & \bigcirc \\ G \\ X_{01} & X_{11} \\ X_{01} & X_{11} \end{bmatrix} = \begin{bmatrix} \bigcirc \\ K_{10} \\ K_{10} \end{bmatrix} + \begin{bmatrix} X_{00} \\ K_{10} \end{bmatrix} + \begin{bmatrix} X_{00} \\ K_{10} \end{bmatrix} + \begin{bmatrix} X_{00} \\ K_{10} \end{bmatrix} + \begin{bmatrix} X_{10} \\ K_{10$$

Rule-based representation

$$\begin{bmatrix} + \bigcirc \rightleftharpoons \begin{bmatrix} \bigcirc & X(a) + A(x) < ->X(a!1).A(x!1) \\ \vdots & + \bigcirc \rightleftharpoons \begin{bmatrix} \bigcirc & X(b) + B(x) < ->X(b!1).B(x!1) \end{bmatrix}$$

Rules representing molecular interactions allow for compact model specifications

The number of rules scales linearly with the number of molecular interactions in a system

Science's STKE re6 (2006)

Network size increases nonlinearly when an extra interaction is considered

Early events in EGFR signaling - we'll consider these events to illustrate modeling approaches



- 1. EGF binds EGFR
- 2. EGFR dimerizes



- 1. EGF binds EGFR
- 2. EGFR dimerizes
- 3. EGFR transphosphorylates itself



Grb2 pathway

- 1. EGF binds EGFR
- 2. EGFR dimerizes
- 3. EGFR transphosphorylates itself
- 4. Grb2 binds phospho-EGFR



Grb2 pathway

- 1. EGF binds EGFR
- 2. EGFR dimerizes
- 3. EGFR transphosphorylates itself
- 4. Grb2 binds phospho-EGFR
- 5. Sos binds Grb2 (Activation Path 1)



- 1. EGF binds EGFR
- 2. EGFR dimerizes
- 3. EGFR transphosphorylates itself
- 4. Shc binds phospho-EGFR



- 1. EGF binds EGFR
- 2. EGFR dimerizes
- 3. EGFR transphosphorylates itself
- 4. Shc binds phospho-EGFR
- **5. EGFR transphosphorylates Shc**



- 1. EGF binds EGFR
- 2. EGFR dimerizes
- 3. EGFR transphosphorylates itself
- 4. Shc binds phospho-EGFR
- 5. EGFR transphosphorylates Shc
- 6. Grb2 binds phospho-Shc



- 1. EGF binds EGFR
- 2. EGFR dimerizes
- 3. EGFR transphosphorylates itself
- 4. Shc binds phospho-EGFR
- 5. EGFR transphosphorylates Shc
- 6. Grb2 binds phospho-Shc
- 7. Sos binds Grb2 (Activation Path 2)



Representation of molecules in a simple model of early events in EGFR signaling



EGFR(1,d,Y1092~U~P,Y1172~U~P)

Blinov et al. (2006)











Dimeric species



A reaction-scheme diagram



This scheme can be translated to obtain a set of ODEs, one for each species

A conventional model for EGFR signaling



*J. Biol. Chem. 274, 30169 (1999)

Assumptions made to limit combinatorial complexity



Assumptions made to limit combinatorial complexity



Outline

- 1. The biochemistry of cell signaling and combinatorial complexity
- 2. The conventional approach to modeling
- 3. The rule-based approach to modeling
- 4. Tools
- 5. New simulation methods

Rules operate on structured objects (graphs)

Graphs represent molecules, their component parts, and states

- A (graph-rewriting) rule specifies the addition or removal of an edge to represent binding or unbinding, or the change of a state label to represent, for example, post-translational modification of a protein at a particular site
- A model specification is readily visualized and compositional
- Molecules, components, and states can be directly linked to annotation in databases





Ty Thomson (MIT) - yeastpheromonemodel.org

Proteins in a model are introduced with molecule templates

Molecule templates



Nodes represent components of proteins

Components may have attributes: o or P

Complexes are connected instances of molecule templates

An EGFR dimer



Edges represent bonds between components

Bonds may be internal or external

Patterns select sets of chemical species with common features

Pattern that selects EGFR phosphorylated at Y1092.



BioNetGen language provides explicit representation of molecules and interactions

Molecules are structured objects (hierarchical graphs)



Rules define interactions (graph rewriting rules)



BNGL: $A(b) + B(a) < -> A(b!1) \cdot B(a!1) \cdot kp1, km1$

a bond between two components

Faeder et al., Proc. ACM Symp. Appl. Computing (2005)

Rules generate events

Example of reaction generation:



Reaction rules, composed of patterns, generalize reactions



Patterns select reactants and specify graph transformation

- Addition of bond between EGF and EGFR

Rule-based version of the Kholodenko model

- 5 molecule types
- 23 reaction rules
- No new rate parameters (!)



Dimerization rule eliminates previous assumption restricting breakup of receptors

EGFR dimerizes (600 reactions)



Dimers form and break up independent of phosphorylation of cytoplasmic domains

Outline

- 1. The biochemistry of cell signaling and combinatorial complexity
- 2. The conventional approach to modeling
- 3. The rule-based approach to modeling

4. Tools

5. New simulation methods

BioNetGen2: Software for graphical rule-based modeling



BNGL: A textual language for graphical rules



L(r) + R(l,d) <-> L(r!1).R(l!1,d) kp1, km1

BNGL: A textual language for graphical rules



Graphical Interface to BioNetGen



Greatly simplifies construction, visualization, and simulation of complex models

We can take advantage of collective intelligence to build large-scale models

One model currently under construction incorporates approximately 20 proteins involved in EGFR signaling (EGF, HRG, EGFR, ErbB2, ErbB3, ErbB4, Shc, Grb2, Sos1, Gab1, PI3K, Akt, Ras, Raf, MEK, ERK)

And approximately 1,000 annotated rules capturing the sitespecific details of protein-protein interactions



Outline

- 1. The biochemistry of cell signaling and combinatorial complexity
- 2. The conventional approach to modeling
- 3. The rule-based approach to modeling
- 4. Tools
- **5.** New simulation methods

Rule-based models can be difficult to simulate

Rule-based models may encompass a large or even an unbounded number of species

- Computational costs for standard simulation methods increase with number of species and reactions in a model
- Parameter estimation and data fitting require running model simulations for a large number of parameter sets
- We need a simulation method that is independent of the size of the reaction network implied by rules

The system: interaction of a trivalent ligand with a bivalent cell-surface receptor



Rule-based model specification corresponding to equilibrium model of Goldstein and Perelson (1984)



Generate-first method of simulation

- 1. Define seed species
- 2. Determine if a pattern in a rule matches any species If so, apply the transformation defined in the rule
- 3. Iteratively apply rules to new product species
- 4. Simulate using conventional methods once network has been generated

Seed species



After first round of rule application



After the second round of rule application



Rule-derived network is too large to simulate using conventional population-based methods



Rule-based KMC Method

(Particle-based version of Gillespie's Direct Method with rules)

- 1. Instantiate molecules with components and states.
- 2. Determine cumulative rate for each reaction type, a_m
- 3. Select next reaction time,

$$\Delta t = -\ln(r_1) / a_{\rm tot}$$

4. Select next reaction type using

$$\sum_{j=1}^{J-1} a_j < r_2 a_{\text{tot}} \le \sum_{j=1}^J a_j$$

- 5. Select reactant molecules and components.
- 6. Update reaction type rates. Iterate.

Kinetics of aggregate formation



Performance

