1 Consider the following two reactions:

$$mRNA \xrightarrow{k_1} mRNA + P \qquad (1)$$
$$mRNA \xrightarrow{k_2} \phi \qquad (2)$$
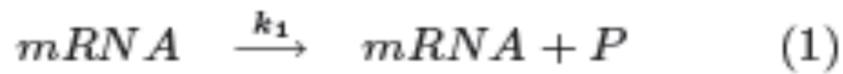
Starting at $t = 0$ with a single mRNA molecule, answer the following questions:

(a) Find the probability that reaction (2) fires before reaction (1) does.

(b) Find the probability that reaction (1) fires at least once before reaction (2) does.

(c) Find the probability that reaction (1) fires exactly n times before reaction (2) does.

2 A transcription factor binds to a gene on two possible sites, creating three different possible configurations for the gene (see figure). If the gene starts at state G1 at time $t = 0$, find an expression for the probability that the gene is in state G3 at time t. Use the transition rates given in the figure.
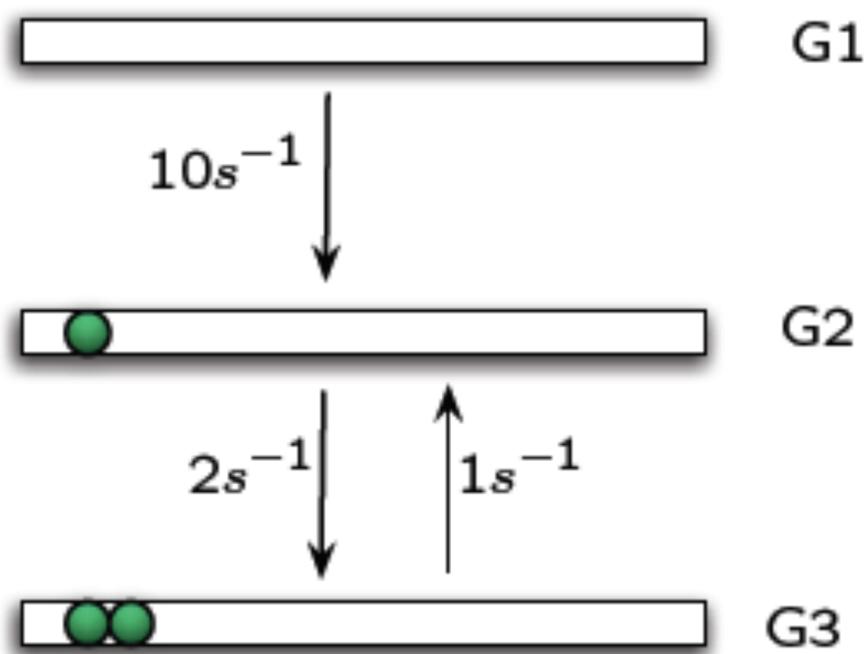
3. You observe a protein that is normally absent, but has occasional spikes in its population level. Each spike is an average of 500 molecules high, lasts for an average of 1 second, and is separated by an average of 100s. Using only the simple transcription and translation model discussed in class:

$$\emptyset \xrightarrow{k_r} mRNA$$
$$mRNA \xrightarrow{\gamma_r} \emptyset$$
$$mRNA \xrightarrow{k_p} mRNA + P$$
$$P \xrightarrow{\gamma_p} \emptyset$$
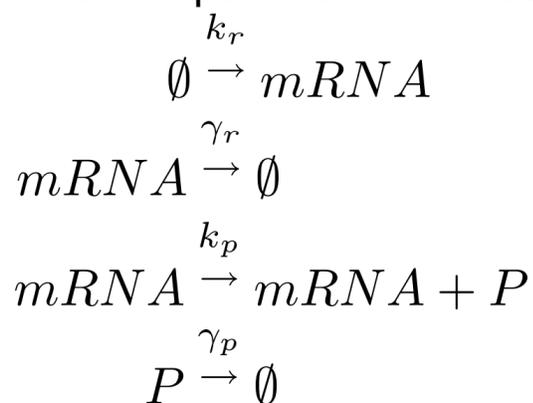
Find a set of parameters $\{k_r, \gamma_r, k_p, \gamma_p\}$ that could account for these observations. Note that this parameter set is not necessarilly unique.